Efficient Hybrid Mode-Matching/-Finite-Element (MM/FE) Method for the Design of Waveguide Components and Slot Radiators

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Abstract— **A hybrid mode-matching/finite-element (MM/FE) method is introduced for the rigorous design and optimization of waveguide components and radiators of arbitrary cross-section. The FE formulation for general cylindrical and conical waveguides, respectively, leads to a fast and direct sparse matrix eigenvalue procedure. Based on the MM technique in combination with a spherical wave expansion for the free space, a system of equations of low order for the coefficients of incident and scattered waves in the feeding waveguide is obtained by which the generalized scattering matrix as well as near- and far-field patterns can be calculated. The hybrid method combines advantageously the flexibility of the FE method with the efficiency of the MM technique. Its versatility is demonstrated at the optimum design of a septum polarizer which is directly combined with a conical circular waveguide horn, and the optimization of a a slot-array element. The theory is verified by measurements.**

I. Introduction

FOR the analysis of more complicated waveguide structures, space discretization methods are typically used, such as the three-dimensional finite element or finite difference time domain methods. Because of the rather high requirements concerning storage capacity and CPU time, however, for the CAD of such structures, adequate hybrid methods are desirable which utilize as much as possible the efficiency of the mode-matching method. A combined mode matching/finite element method has been proposed for the design of dual-mode filters and waffle-iron filters in [1], [4]. In [2], a hybrid boundary contour mode matching method with spherical modes is applied for the modal analysis of arbitrarily shaped waveguide structures with symmetry of revolution.

In this paper, the hybrid mode-matching/finite-element method is extended to include spherical waveguide regions both for inner and outer structures. The cylindrical and conical waveguide eigenvalue problem of cross-sections of arbitrary shape is solved by the FE method. An appropriate Riemann-like unit sphere coordinate transformation for the conical waveguide problem into an u, v-plane advantageously allows the Cartesian coordinate FE program code to be applied with merely minor modifications.

The mode-matching technique formulated for the discontinuity between conical waveguide and spherical wave region leads to a real and frequency independent coupling matrix. A reciprocal field expansion at the common interface solves the discontinuity problem between the conical and cylindrical waveguides. For the resulting system of equations, the spherical and conical wave coefficients can be eliminated without any inversion which yields a very fast algorithm for the frequency loop. The derived generalized scattering matrix (GSM) can be easily combined with standard waveguide mode-matching building blocks. The near- and far-field patterns are obtained by the resulting spherical wave coefficients.

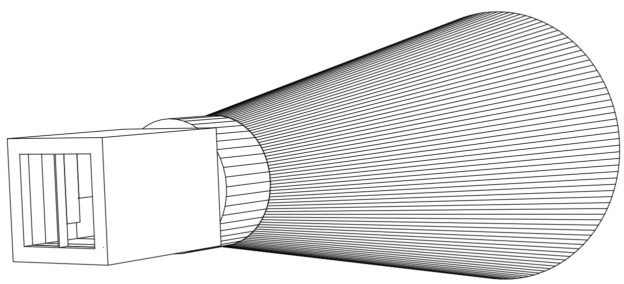

Fig. 1. Septum polarizer connected to a conical circular waveguide horn.

The efficiency of the method is demonstrated by the optimized design of a septum polarizer which is connected directly to a conical circular waveguide horn antenna (Fig. 1). This leads to a symmetric, circular polarized field pattern with low cross polarization. The flexibility of the method is illustrated by the successful simulation of a slot array radiator (Fig. 2) [3]. For this structure excellent agreement between theory and

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measurements can be stated.

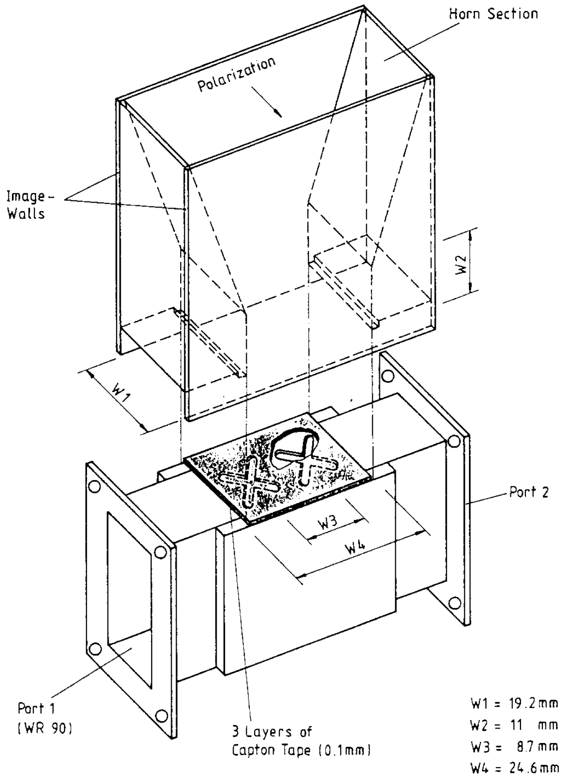


Fig. 2. Slot array radiating element simulated by the advanced MM/FE method.

II. Theory

For structures which require a high number of eigenmodes to be considered and which may include sharp edges, like the septum polarizer elements in Fig. 1, the hybrid MM/FE method [1], [4] is applied because of its high flexibility.

In contrast to usual other methods, all desired eigenvalues and corresponding eigenvectors are computed directly by an efficient sparse matrix eigenvalue procedure for cross sections of arbitrary shape. This fact makes the hybrid MM/FE method very suitable for the fast CAD of more general waveguide components which require the consideration of a large number of higher-order modes. The conical waveguide modes can easily be computed with the same algorithm as for cylindrical waveguides by application of a Riemann-like coordinate transformation which maps the unit sphere into an u, v-plane.

For a conical waveguide with propagation axis in ϑ_0, φ_0 - direction related to a global spherical coordinate system, the transformation is given by:

$$\begin{aligned} u' &= \frac{\sin\vartheta\cos\vartheta_0\cos(\varphi-\varphi_0)-\cos\vartheta\sin\vartheta_0}{1+\cos\vartheta\cos\vartheta_0+\sin\vartheta\sin\vartheta_0\cos(\varphi-\varphi_0)} \\ v' &= \frac{\sin\vartheta\sin(\varphi-\varphi_0)}{1+\cos\vartheta\cos\vartheta_0+\sin\vartheta\sin\vartheta_0\cos(\varphi-\varphi_0)}, \\ u &= u'\cos\xi_0 + v'\sin\xi_0, \\ v &= -u'\sin\xi_0 + v'\cos\xi_0; \end{aligned} \tag{1}$$

this is illustrated in Fig. 3.

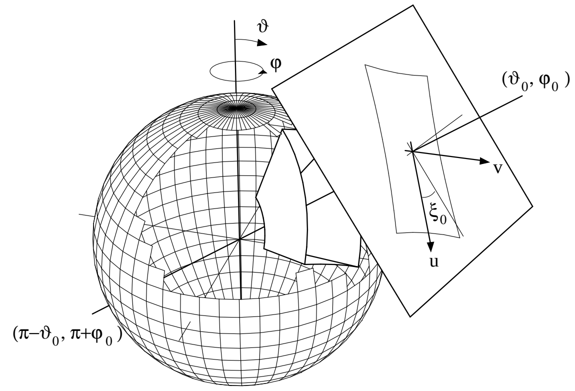


Fig. 3. Coordinate transformation for radiating conical waveguides.

The solution of Maxwell's equations in the new coordinates are based on the following scalar potentials for TE- and TM-waves:

$$\Psi(u,v,\rho) = S(u,v)(A\,\hat{j}_\nu(k\rho) + B\,\hat{y}_\nu(k\rho)), \tag{2}$$

where $S(u,v)$ is a solution of the eigenvalue problem

$$\frac{\partial^2 S}{\partial u^2} + \frac{\partial^2 S}{\partial v^2} + \frac{4\nu(\nu+1)S}{(1+u^2+v^2)^2} = 0 \tag{3}$$

and $\hat{j}_\nu, \hat{y}_\nu$ are the Riccati-Bessel and Neumann functions of ν-th order.

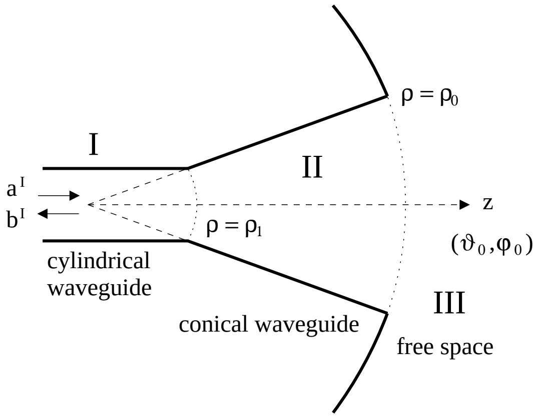


Fig. 4. Computational model for radiating conical waveguides.

The electric $\vec{E}$ and magnetic fields $\vec{H}$ in the free space or in the spherical resonator (region III in Fig. 4) are represented by the usual spherical wave expansion [2], by solutions of the transverse homogeneous Helmholtz equation in the cylindrical (I) waveguide sections [1], [4], and by solutions $S(u,v)$ of (3) in the conical waveguide section (II). The solutions of (3) satisfy Dirichlet and Neumann boundary conditions on perfectly conducting electric and magnetic walls, and show the same orthonormal properties as cylindrical waveguide modes.

The application of the mode-matching method at the common interfaces between free space region and conical waveguide (regions II and III) at $\rho = \rho_0$ leads to a real and frequency independent coupling matrix $\mathbf{K}$ like for usual cylindrical waveguide step discontinuities. The continuity of the transverse electric and magnetic fields at the interface between regions I and II ($\rho = \rho_1$) is achieved by a reciprocal modal Galerkin procedure which results in 4 different coupling matrices $\mathbf{C}_a^E$ $\mathbf{C}_b^E$ $\mathbf{C}_a^H$ $\mathbf{C}_b^H$ for the incident and scattered waves where the superscripts denote the $E-$ or $H-$ field continuity, respectively.

The unknown wave coefficients of regions II and III can be eliminated without any inversion. This leads to the final system of equations for the coefficients of the incident and scattered waves a^I, b^I:

$$\begin{aligned}&\mathbf{P}_1(\mathbf{C}_a^H a^I + \mathbf{C}_b^H b^I) + \mathbf{P}_3(\mathbf{C}_a^E\, a^I + \mathbf{C}_b^E b^I) = \\ &\mathbf{K}^T\mathbf{Y}\mathbf{K}[\mathbf{P}_2(\mathbf{C}_a^E a^I + \mathbf{C}_b^E\, b^I) + \mathbf{P}_4(\mathbf{C}_a^H a^I + \mathbf{C}_b^H\, b^I)].\end{aligned} \tag{4}$$

In this final equation, $\mathbf{P}_{1-4}$ and $\mathbf{Y}$ denote diagonal matrices with cross-products of Riccati-Bessel functions and normalized spherical field admittances for $\rho = \rho_0$.

Note, that both CPU time and memory requirements depend only linear on the number of spherical waves for region III, which can be quite high (up to 10000) for large spheres ($\rho_0 \gg \lambda$) and asymmetric structures with $\vartheta_0 \neq 0$. On the other hand, only a few number of waveguide modes (the guided and some lower order evanescent modes) have to been considered in this formulation, so the computational effort in the frequency loop is relatively small.

The mesh for the two-dimensional FEM solution of the Helmholtz equation for the sections with arbitrary geometry is generated by a conforming Delaunay triangulation. For the conical waveguides the triangulation is carried out directly in the u,v-plane, the related Cartesian mesh is obtained by a point-wise coordinate transformation.

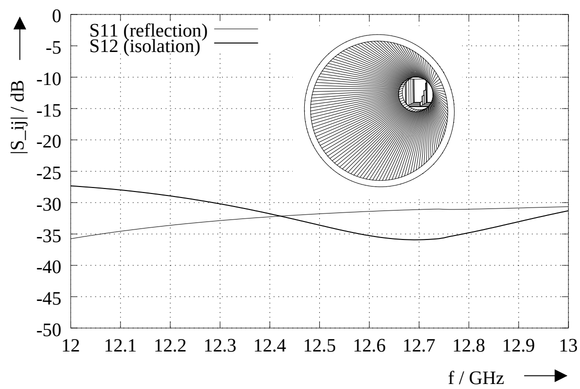
The generalized sparse matrix eigenvalue problem resulting from the FE problem is reduced to tridiagonal form by the Lanczos procedure, with application of a shift and invert technique to accelerate convergence. Full Gram-Schmidt type re-orthogonalization guarantees the orthogonality of even higher-order multiple degenerate modes. The system of equations arising in each Lanczos iteration step is solved by sparse matrix Cholesky decomposition using the minimum degree algorithm.

The surface integrals over the aperture for the coupling matrices $\mathbf{K}$ and $\mathbf{C}_{a,b}^{E,H}$ are evaluated on the u,v-plane FE-mesh using a seven point integration rule.

With the knowledge of a^I and b^I, the spherical wave coefficients of region III for each incident wave are be obtained by a simple matrix-vector multiplication. The resulting GSM-formulation can be combined easily with other (standard) waveguide building blocks. Far- and near-field patterns are computed from the resulting spherical wave coefficients after application of the GSM combination.

III. Results

The first example is an optimized square septum polarizer connected to a conical circular waveguide horn. Figs. 5 show the return loss, isolation and the resulting circular polarized far-field gain-pattern of the structure at 12.5 GHz.



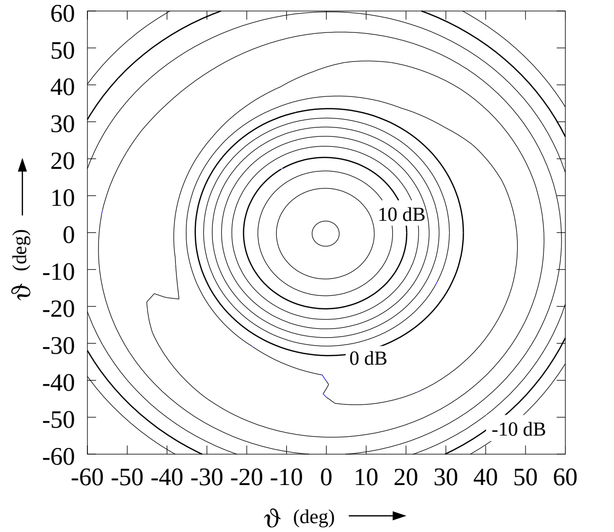
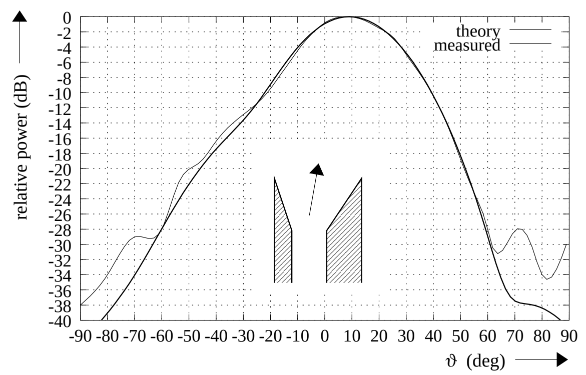
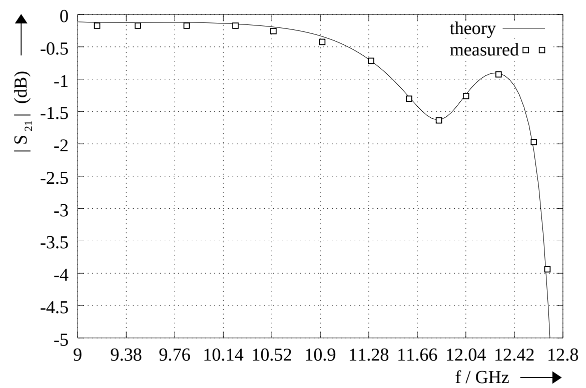


Fig. 5. Scattering parameters (a) and gain pattern at 12.5 GHz (b) of the optimized square septum polarizer according to Fig. 1 including the circular waveguide horn.

In order to demonstrate the flexibility of the presented method, we calculated a slot radiator element (Fig. 3) according to [3]. The asymmetric H-plane horn is coupled to a T-junction via a two-slot iris, covered with a thin layer of capton tape included in the simulation. A rectangular conical waveguide with zero length is used as intermediate medium in the radiation model.





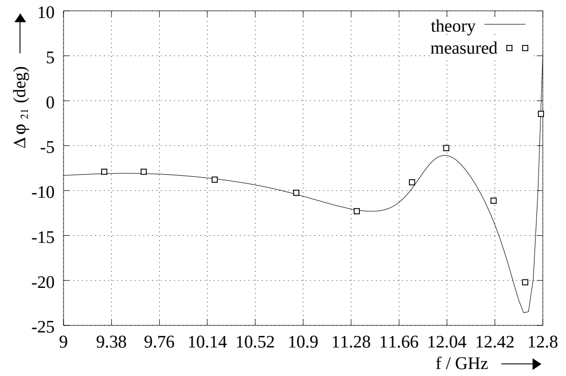


Fig. 6. Slot radiator element (Fig. 3) [3]. (a) H-plane pattern at 10.9 GHz (b) Measured and computed transmission coefficient $|S_{21}|$ (c) Measured and computed transmission phase $\Delta\varphi_{21}$ (related to an empty WR90 waveguide of equal length).

The H-plane pattern of the slot radiator, as well the transmission coefficient in the feeding waveguide are shown in Figs. 6. Excellent agreement with measured data may be stated.

IV. Conclusion

An extended hybrid mode-matching/finite-element (MM/FE) method is presented. The combination with a spherical wave expansion allows both to include spherical waveguide structures and radiating apertures. The hybrid method combines advantageously the flexibility of the FE method with the efficiency of the MM technique, and, hence, allows the convenient optimization of the structures.

Acknowledgment

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